A viscoelasticity – viscoplasticity material model for superalloy applications

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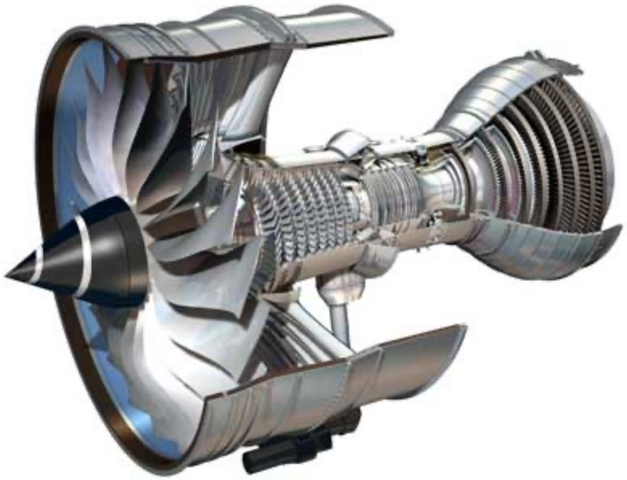
This project has received funding from the *European Union's Horizon 2020 research and innovation programme* and Joint Undertaking Clean Sky 2 under grant agreement No 686600.

Introduction

Motivation

- The European ACARE 2050 strategic agenda sets out ambitious goals to reduce CO2 and NOx emissions and perceived noise (75%, 90%, and 65%, respectively) by the year 2050.
- At present 3 Gtonnes of CO2 are produced every year by air travel. This is completely unsustainable and is driving the need for greater efficiency in aeroengines.
- Future jet engine designs are expected to utilise higher core temperatures and lower component weights in order to meet these targets.
- Turbine discs (RR1000) are already subjected to rigorous lifing assessments however the range of applicability needs to be extended as a results of these shifts in design paradigms.



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DevTMF

Development of Experimental Techniques and Predictive Tools to Characterise Thermo-Mechanical Fatigue Behaviour and Damage Mechanisms (DevTMF)

- **Dev**elopment of Experimental Techniques and Predictive Tools to Characterise **T**hermo-**M**echanical **F**atigue Behaviour and Damage Mechanisms
- An EU (H2020) funded collaborative project between Rolls-Royce, Linkoping University, Swansea University and the University of Nottingham.
- The project aims to increase operational and service life of present and future gas turbine components by enabling more accurate predictions of design life.
- Nottingham contacts for the project are Dr. C. J. Hyde (lead) and Dr. J. P. Rouse (deputy lead).

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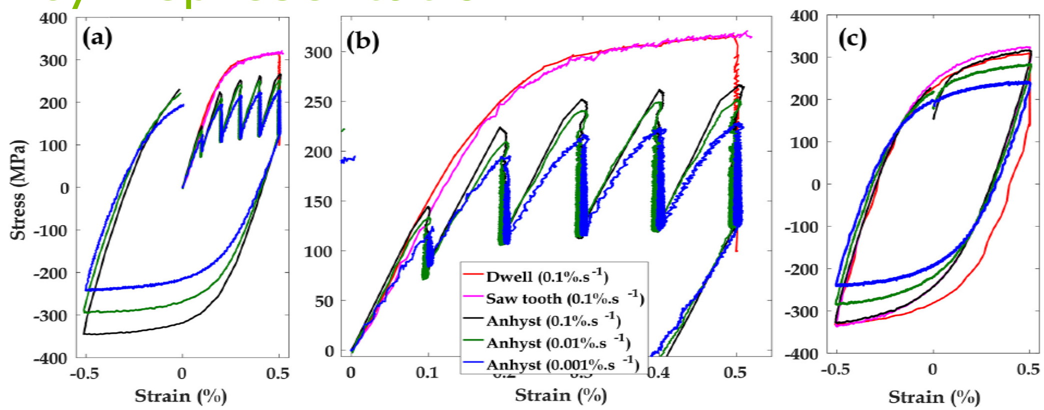
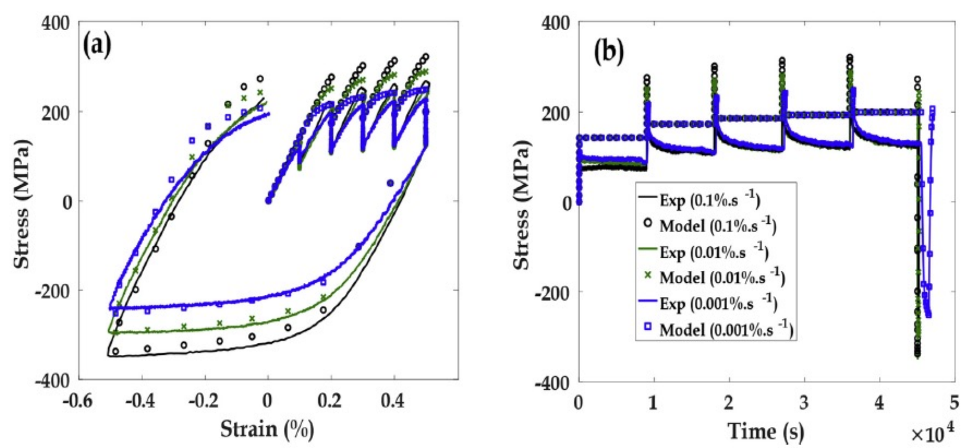
Elastic – Viscoplastic Model

Limitations in Time Dependency Representation

- It is well known that materials, such as the P91 chromium steel shown here, will exhibit time dependencies when loaded at elevated temperature.
- Elastic - viscoplastic model formulations are very common, however necessarily impose restrictions of the degree to which stresses can relax.
- The present work looks to develop a viscoelastic – viscoplastic material model from a thermodynamic basis. Two sinh flow rules are defined such that a wide range of time dependencies can be captured.





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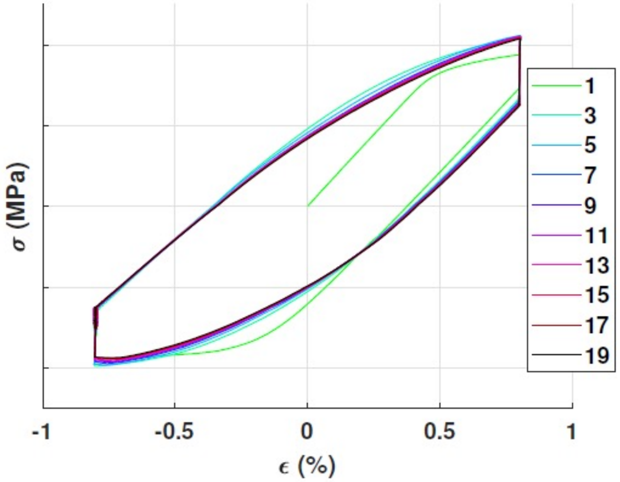
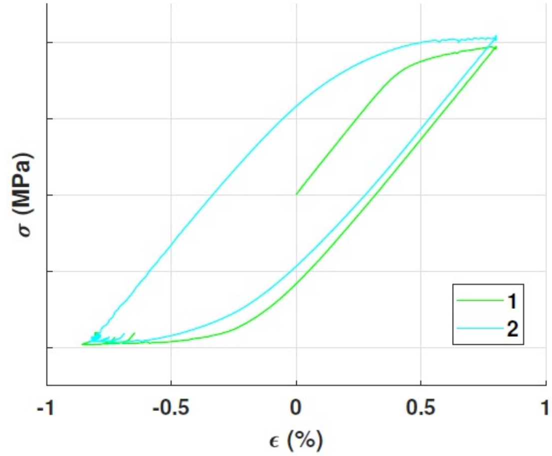
Experimental Data

Experimental Observations for RR1000

- To illustrate the applicability of the model, all parameter calibration is performed using a limited number of experimental results that used standard waveforms.
- Attention is given to the description of behaviours at 750°C in the RR1000 material as little to no stress relaxation is observed at 400°C.





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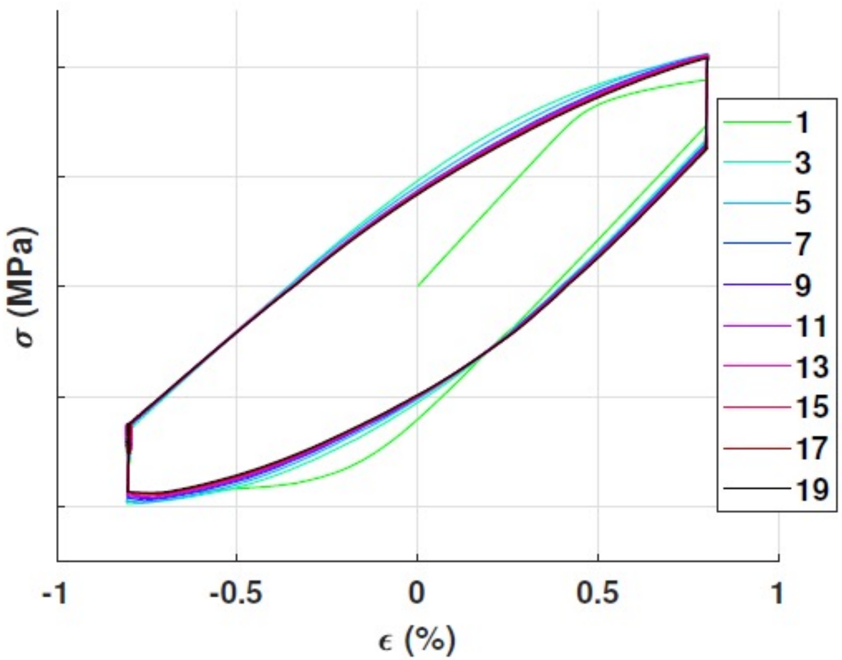

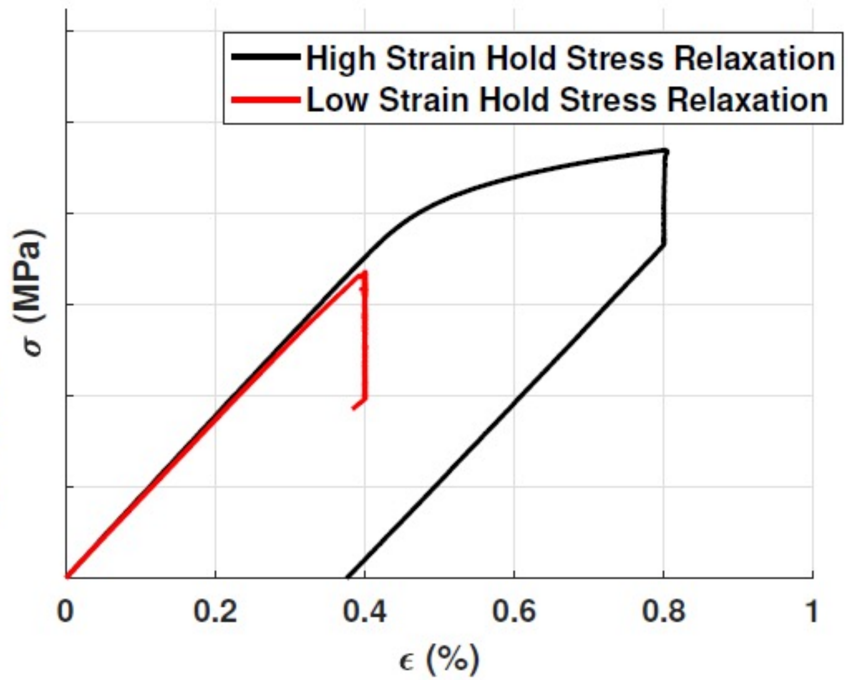

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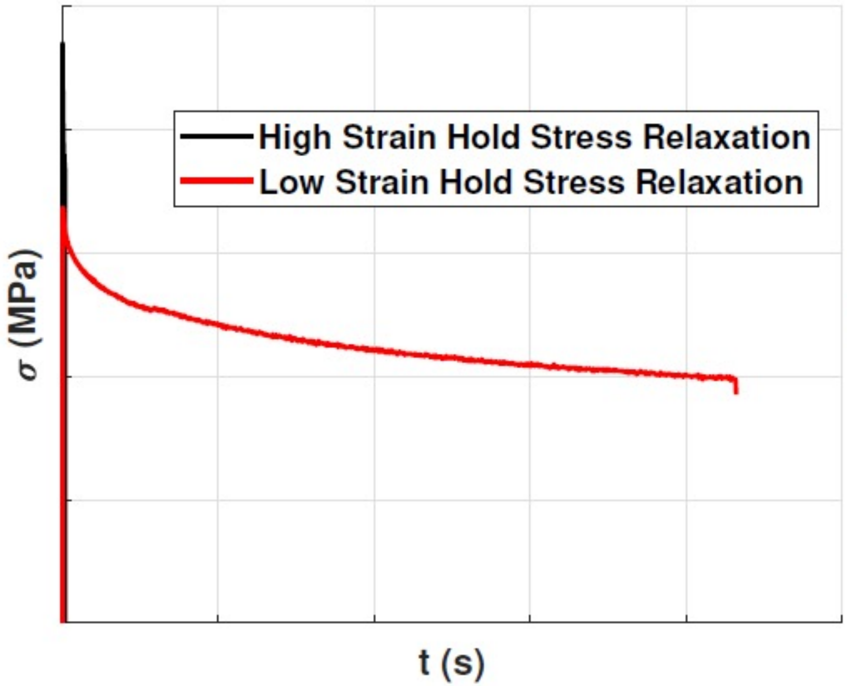

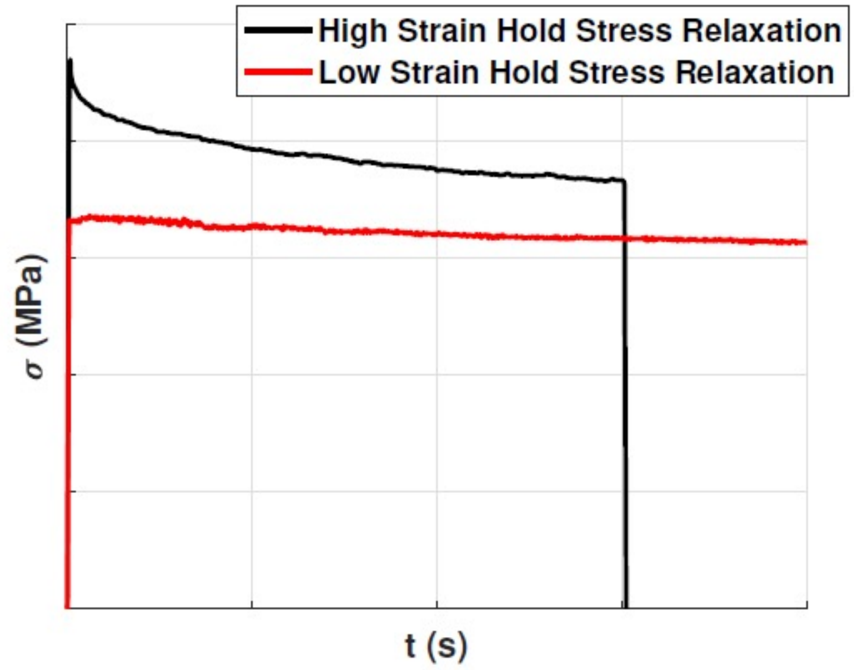

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Material Model Formulation

Viscoelastic - Viscoplastic Model

- The viscoelastic – viscoplastic material model developed here is thermodynamically based using the thermodynamics of irreversible processes formalism.
- Helmholtz free energy and dual dissipation potential functions are defined such that state laws and evolution equations can be determined in the usual way.
- While a general version of the model is developed in the multiaxial case, several simplifying conditions are applied in the uniaxial case to limit the required number of parameters to 15.

$$\psi = \frac{1}{2}(\epsilon - \epsilon_{VE} - \epsilon_{VP}) : C_e : (\epsilon - \epsilon_{VE} - \epsilon_{VP}) + \frac{1}{3}\sum_{i=1}^{n_{VE}} C_i^{VE} \alpha_i^{VE} : \alpha_i^{VE} + \ldots$$

$$\frac{Q^{VE}}{b^{VE}}\left(b^{VE} r^{VE} + \exp\left(-b^{VE} r^{VE}\right)\right) + \frac{1}{3}\sum_{i=1}^{n_{VP}} C_i^{VP} \alpha_i^{VP} : \alpha_i^{VP} + \frac{Q^{VP}}{b^{VP}}\left(b^{VP} r^{VP} + \exp\left(-b^{VP} r^{VP}\right)\right)$$

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Material Model Formulation

Viscoelastic - Viscoplastic Model

$$\psi = \frac{1}{2}(\epsilon - \epsilon_{VE} - \epsilon_{VP}) : C_e : (\epsilon - \epsilon_{VE} - \epsilon_{VP}) + \frac{1}{3}\sum_{i=1}^{nVE} C_i^{VE} \boldsymbol{\alpha}_i^{VE} : \boldsymbol{\alpha}_i^{VE} + \ldots$$

$$\frac{Q^{VE}}{b^{VE}}\left(b^{VE} r^{VE} + \exp\left(-b^{VE} r^{VE}\right)\right) + \frac{1}{3}\sum_{i=1}^{nVP} C_i^{VP} \boldsymbol{\alpha}_i^{VP} : \boldsymbol{\alpha}_i^{VP} + \frac{Q^{VP}}{b^{VP}}\left(b^{VP} r^{VP} + \exp\left(-b^{VP} r^{VP}\right)\right)$$

$$\sigma = \frac{\partial \psi}{\partial \epsilon_e} = C_e : (\epsilon - \epsilon_{VE} - \epsilon_{VP})$$

$$\chi_i^{VE} = \frac{\partial \psi}{\partial \boldsymbol{\alpha}_i^{VE}} = \frac{2}{3} C_i^{VE} \boldsymbol{\alpha}_i^{VE}$$

$$\chi_i^{VP} = \frac{\partial \psi}{\partial \boldsymbol{\alpha}_i^{VP}} = \frac{2}{3} C_i^{VP} \boldsymbol{\alpha}_i^{VP}$$

$$X_{VE} = \frac{\partial \psi}{\partial \epsilon_{VE}} = -\sigma$$

$$X_{VP} = \frac{\partial \psi}{\partial \epsilon_{VP}} = -\sigma$$

$$R^{VE} = \frac{\partial \psi}{\partial r^{VE}} = Q^{VE}\left(1 - \exp\left(-b^{VE} r^{VE}\right)\right)$$

$$R^{VP} = \frac{\partial \psi}{\partial r^{VP}} = Q^{VP}\left(1 - \exp\left(-b^{VP} r^{VP}\right)\right)$$

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Material Model Formulation

Viscoelastic - Viscoplastic Model

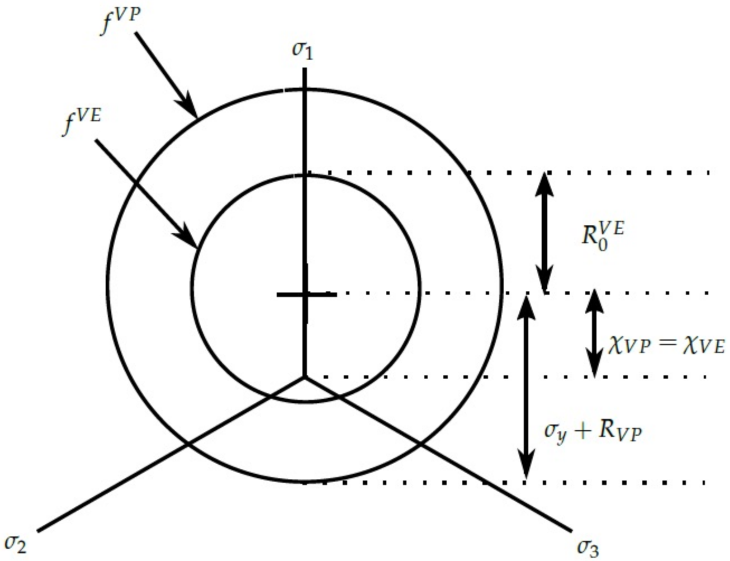
$$f^{VE} = J_2\left(\sigma' - \chi^{VE}\right) - R^{VE} - R_o^{VE}$$

$$f^{VP} = J_2\left(\sigma' - \chi^{VP}\right) - R^{VP} - \sigma_y$$

$$J_2(X) = \left(\frac{3}{2}X : X\right)^{1/2}$$

$$F^{VE} = f^{VE} + \frac{1}{2}\sum_{i=1}^{n_{VE}} \frac{\gamma_i^{VE}}{C_i^{VE}} J_2^2\left(\chi_i^{VE}\right)$$

$$F^{VP} = f^{VP} + \frac{1}{2}\sum_{i=1}^{n_{VP}} \frac{\gamma_i^{VP}}{C_i^{VP}} J_2^2\left(\chi_i^{VP}\right)$$



$$\phi^* = \int A^{VE}\left[\sinh\left(\frac{f^{VE}}{K^{VE}}\right)\right]^{m^{VE}} dF_{VE} + \int A^{VP}\left[\sinh\left(\frac{f^{VP}}{K^{VP}}\right)\right]^{m^{VP}} dF_{VP}$$

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Material Model Formulation

Viscoelastic - Viscoplastic Model

$$\dot{\epsilon}_{VE} = \frac{\partial \phi^*}{\partial X_{VE}} = -\frac{\partial \phi^*}{\partial F_{VE}}\frac{\partial F_{VE}}{\partial X_{VE}} = A^{VE}\left[\sinh\left(\frac{f^{VE}}{K^{VE}}\right)\right]^{m^{VE}} N_{VE} = \dot{\lambda}_{VE} N_{VE}$$

$$\dot{\epsilon}_{VP} = \frac{\partial \phi^*}{\partial X_{VP}} = -\frac{\partial \phi^*}{\partial F_{VP}}\frac{\partial F_{VP}}{\partial X_{VP}} = A^{VP}\left[\sinh\left(\frac{f^{VP}}{K^{VP}}\right)\right]^{m^{VP}} N_{VP} = \dot{\lambda}_{VP} N_{VP}$$

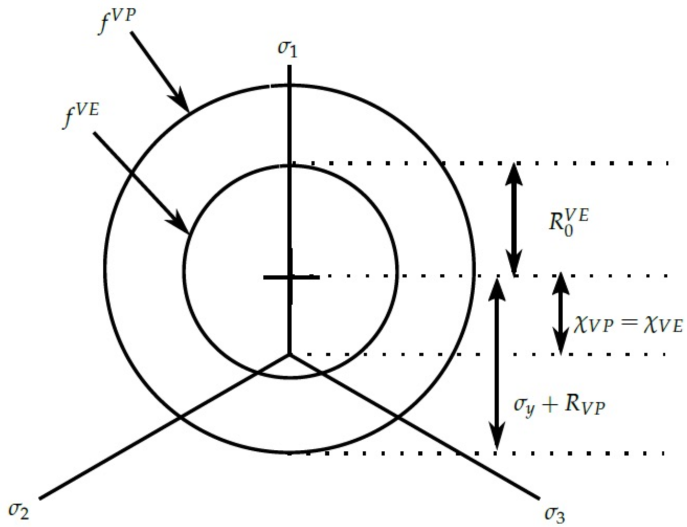
$$\dot{\alpha}_i^{VE} = \frac{\partial \phi^*}{\partial \chi_{VE}} = -\frac{\partial \phi^*}{\partial F_{VE}}\frac{\partial F_{VE}}{\partial \chi_{VE}} = -\dot{\lambda}_{VE}\left(-\frac{3}{2}\frac{(\sigma' - \chi_{VE})}{J_2(\sigma' - \chi_{VE})} + \frac{3}{2}\frac{\gamma_i^{VE}}{C_i^{VE}}\chi_i^{VE}\right)$$

$$= \left[N_{VE} - \frac{3}{2}\frac{\gamma_i^{VE}}{C_i^{VE}}\chi_i^{VE}\right]\dot{\lambda}_{VE}$$

$$= \dot{\epsilon}_{VE} - \frac{3}{2}\frac{\gamma_i^{VE}}{C_i^{VE}}\chi_i^{VE}\dot{\lambda}_{VE}$$

$$\dot{\chi}_i^{VE} = \frac{2}{3}C_i^{VE}\dot{\alpha}_i^{VE} = \frac{2}{3}C_i^{VE}\dot{\epsilon}_{VE} - \gamma_i^{VE}\chi_i^{VE}\dot{\lambda}_{VE}$$

$$\dot{\chi}_i^{VP} = \frac{2}{3}C_i^{VP}\dot{\alpha}_i^{VP} = \frac{2}{3}C_i^{VP}\dot{\epsilon}_{VP} - \gamma_i^{VP}\chi_i^{VP}\dot{\lambda}_{VP}$$



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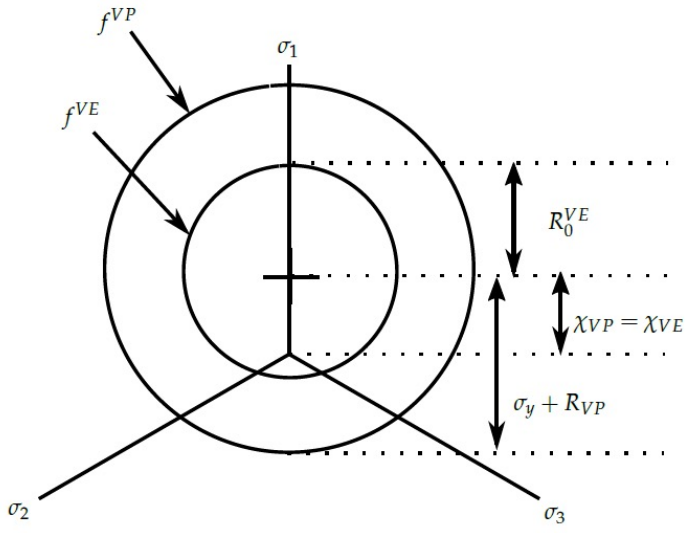
Material Model Formulation

Viscoelastic - Viscoplastic Model

Strain Decomposition:-	$\epsilon = \epsilon_e + \epsilon_{VE} + \epsilon_{VP}$
Stress:-	$\sigma = E\left(\epsilon - \epsilon_{VE} - \epsilon_{VP}\right)$
Viscoelastic Limit Function:-	$f^{VE} = J_2(\sigma - \chi) - R_o^{VE}$
Viscoplastic Yield Function:-	$f^{VP} = J_2(\sigma - \chi) - R - \sigma_y$
Isotropic Hardening (Drag Stress):-	$R = b(Q - R)\lambda_{VP}$
Kinematic Hardening (Back Stress):-	$\chi_i = C_i\dot{\epsilon}_{VP} - \gamma_i\chi_i\dot{\lambda}_{VP}$
Back Stress Decomposition:-	$\chi = \sum_{i=1}^{2}\chi_i$
Viscoelastic Strain Rate:-	$\dot{\epsilon}_{VE} = \left\langle A^{VE}\left[\sinh\left(\frac{f^{VE}}{K}\right)\right]^{m^{VE}}\right\rangle sgn\left(\sigma - \chi\right)$
Accumulated Viscoelastic Strain:-	$\dot{\lambda}_{VE} = \lvert\dot{\epsilon}_{VE}\rvert$
Viscoplastic Strain Rate:-	$\dot{\epsilon}_{VP} = \left\langle A^{VE}\left[\sinh\left(\frac{f^{VE}}{K}\right)\right]^{m^{VE}}\right\rangle sgn\left(\sigma - \chi\right)$
Accumulated Viscoplastic Strain:-	$\dot{\lambda}_{VP} = \lvert\dot{\epsilon}_{VE}\rvert$



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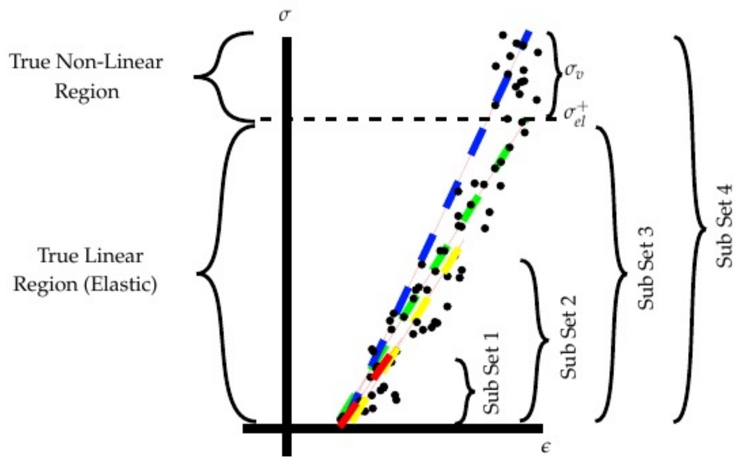
Material Parameter Determination and Optimisation

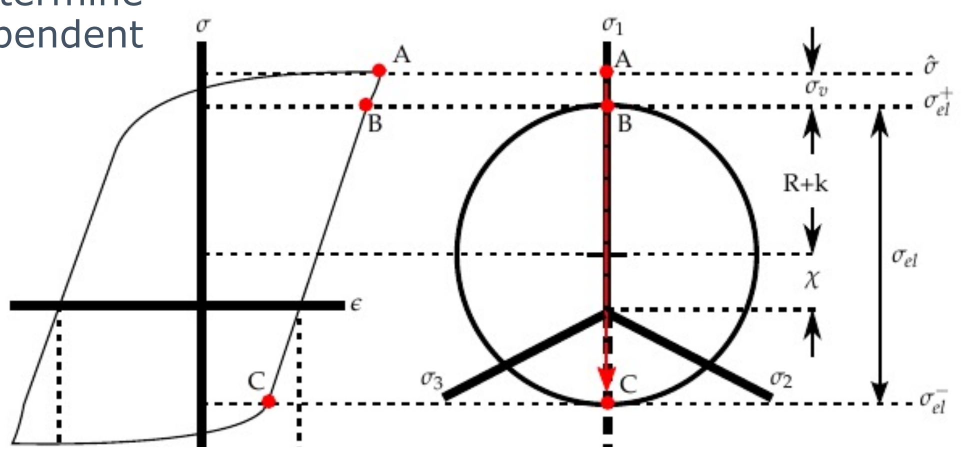
Stress Partitioning and Limits

- Estimates of thermodynamic forces can be made by analysing cyclic data with Cottrell's stress partitioning method.
- Linear regression methods are used to estimate elastic limits in fully reversed experimental hysteresis loops. Cottrell's stress partitioning method is used to approximate dual variable evolution and determine approximate values for material dependent parameters.

$$\sigma_v = \hat{\sigma} - \sigma_{el}^{+} \qquad R + k = \frac{\sigma_{el}}{2}$$

$$\chi = \begin{cases} \sigma_{el}^{+} - \frac{\sigma_{el}}{2}, & \text{if } \sigma_{el}^{+} \geq \sigma_{el}^{-} \\ 0, & \text{if } \sigma_{el}^{+} = \sigma_{el}^{-} \\ \sigma_{el}^{-} + \frac{\sigma_{el}}{2}, & \text{if } \sigma_{el}^{-} \geq \sigma_{el}^{+} \end{cases}$$





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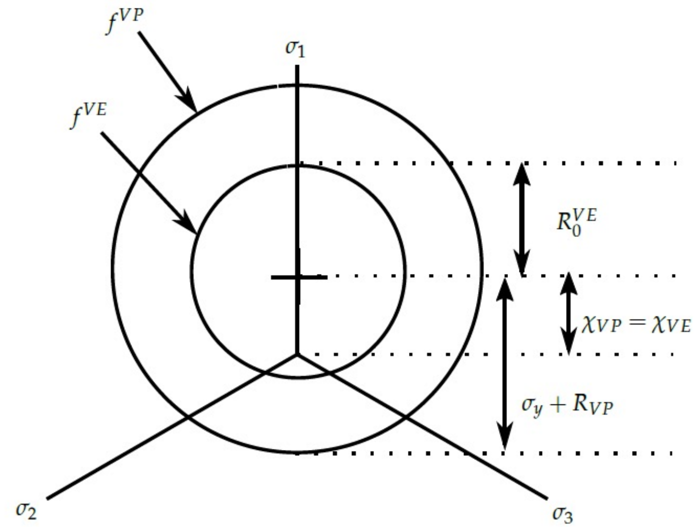
Material Parameter Determination and Optimisation

Stress Partitioning and Limits

- Estimates of material parameters for sinh flow rule terms requires some care as the problem becomes very non-linear and high sensitive to parameter values.
- "Overstresses" for the viscoelastic and viscoplastic cases are estimated from experimental data by assuming a size of the limiting surface.
- Strain rates are approximated from experimental data using the strain decomposition and applying a smoothing function (e.g. a power law).
- By assuming a K value (of similar magnitude to the overstress) a simple linear function can be approximated.

$$\epsilon_{VE} = \epsilon - \epsilon_e = \epsilon - \frac{\sigma}{E}$$

$$\ln(\dot{\epsilon}_{VE}) = \ln\left(A^{VE}\right) + m^{VE}\left(\frac{\sigma_{VE}}{K^{VE}}\right) - m^{VE}\ln(2)$$



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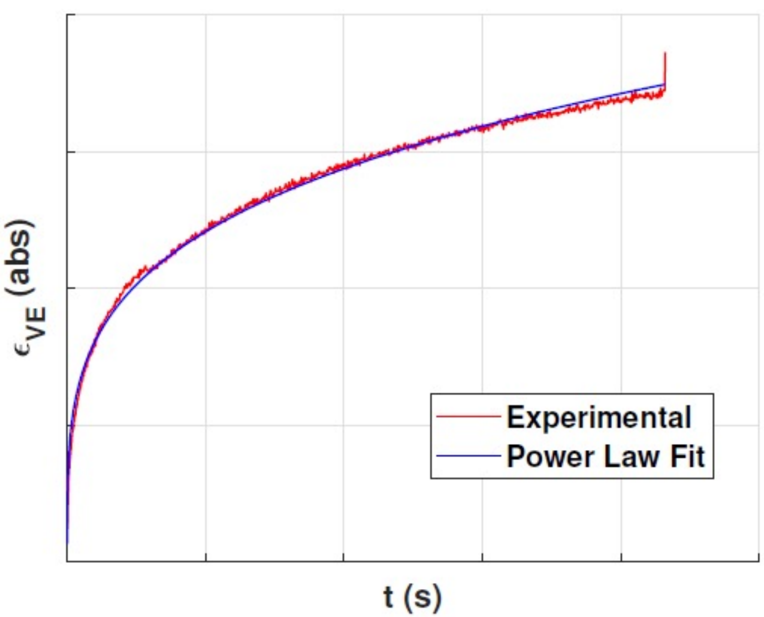

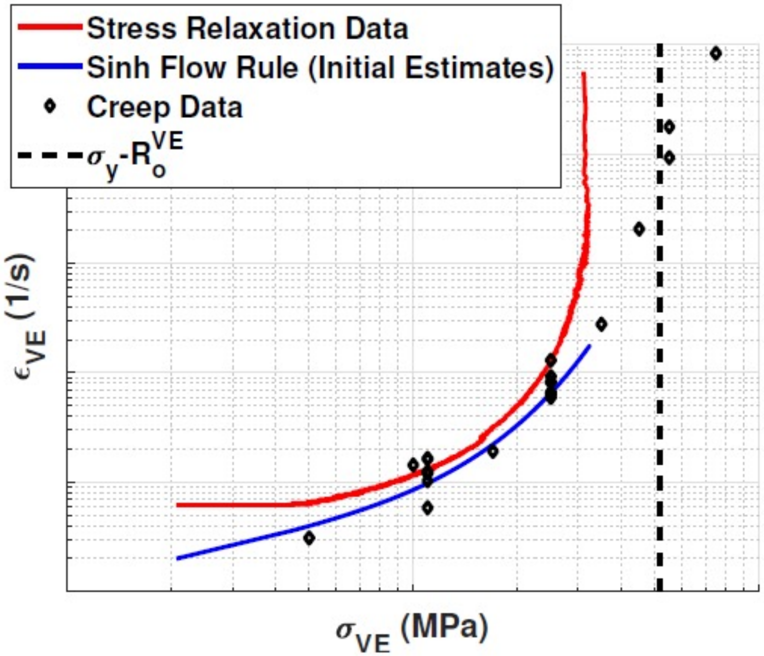

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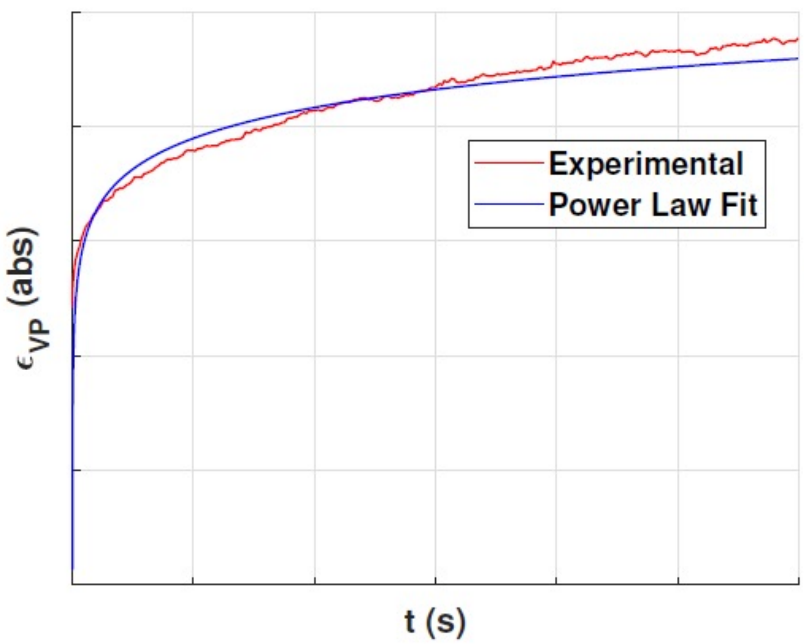

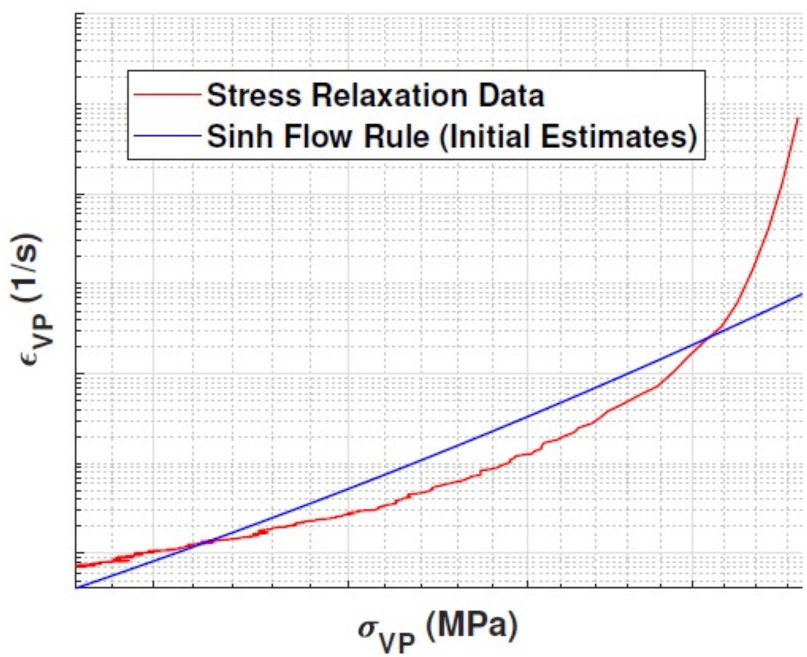

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Material Parameter Determination and Optimisation

Stress Partitioning and Limits

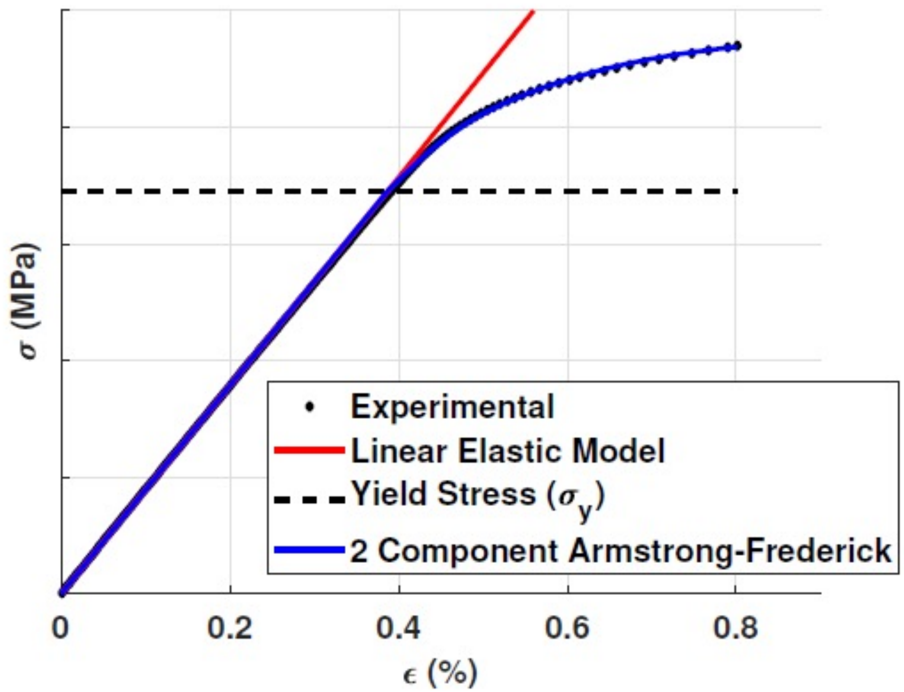
- Armstrong-Frederick parameters are determined using Cottrell stress partition results and through inspection of the monotonic data.
- By approximating a saturation value for the back stresses the approximate relationships outlined below can be applied.

$$\chi_i = \frac{1}{\gamma_i} C_i \left[1 - \exp\left(-\gamma_i \epsilon_p\right)\right]$$

$$\bar{\chi}_i = \frac{C_i}{\gamma_i}$$

$$\tilde{\chi}_i = \frac{C_i}{\gamma_i} \left[1 - \exp\left(-\gamma_i \tilde{\epsilon}_p\right)\right] = \bar{\chi}_i \left[1 - \exp\left(-\gamma_i \tilde{\epsilon}_p\right)\right]$$

$$\gamma_i = -\frac{1}{\tilde{\epsilon}_p} \ln\left(1 - \frac{\tilde{\chi}_i}{\bar{\chi}_i}\right)$$



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Material Parameter Determination and Optimisation

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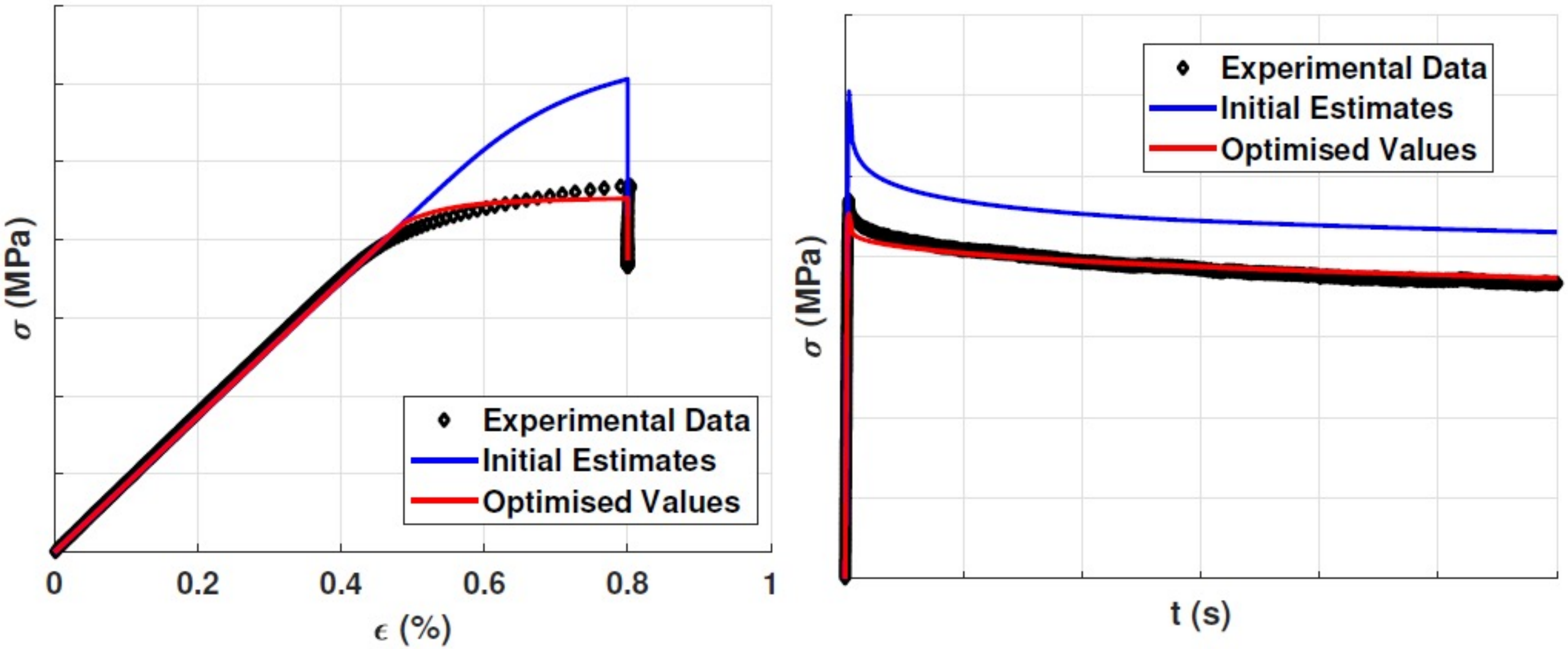

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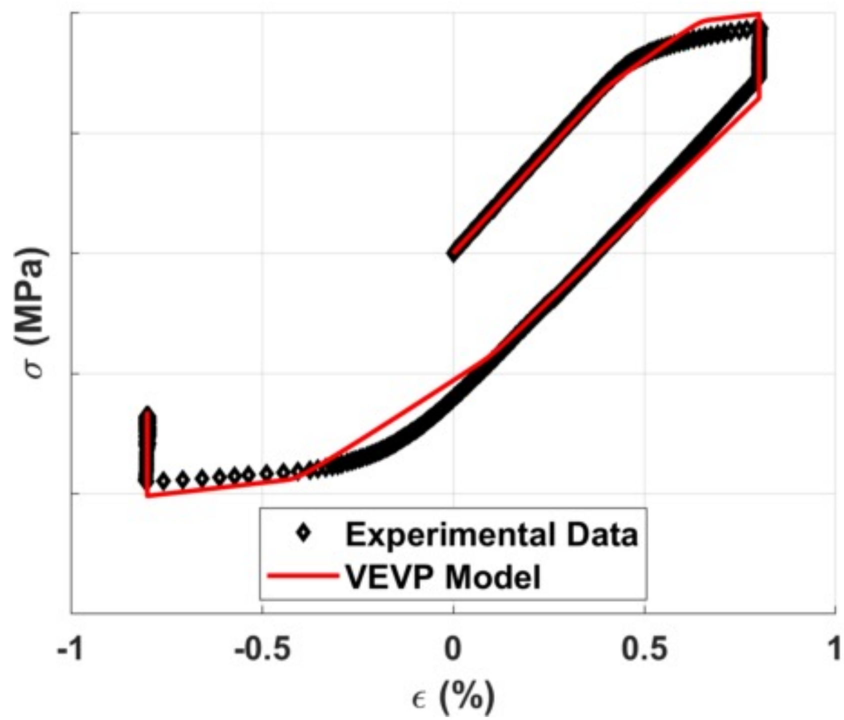
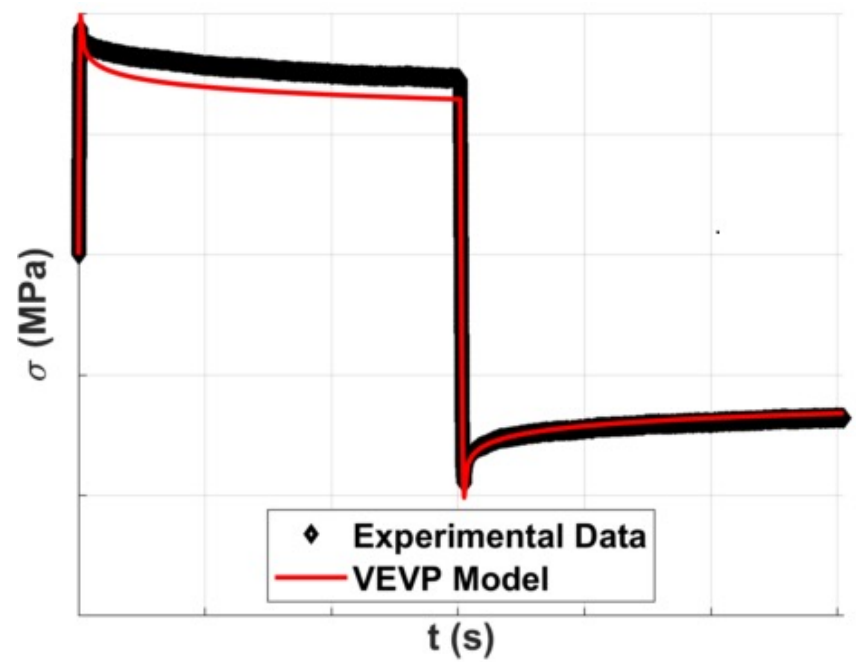
750°C Cyclic Data Prediction

Monotonic and Cycle 1





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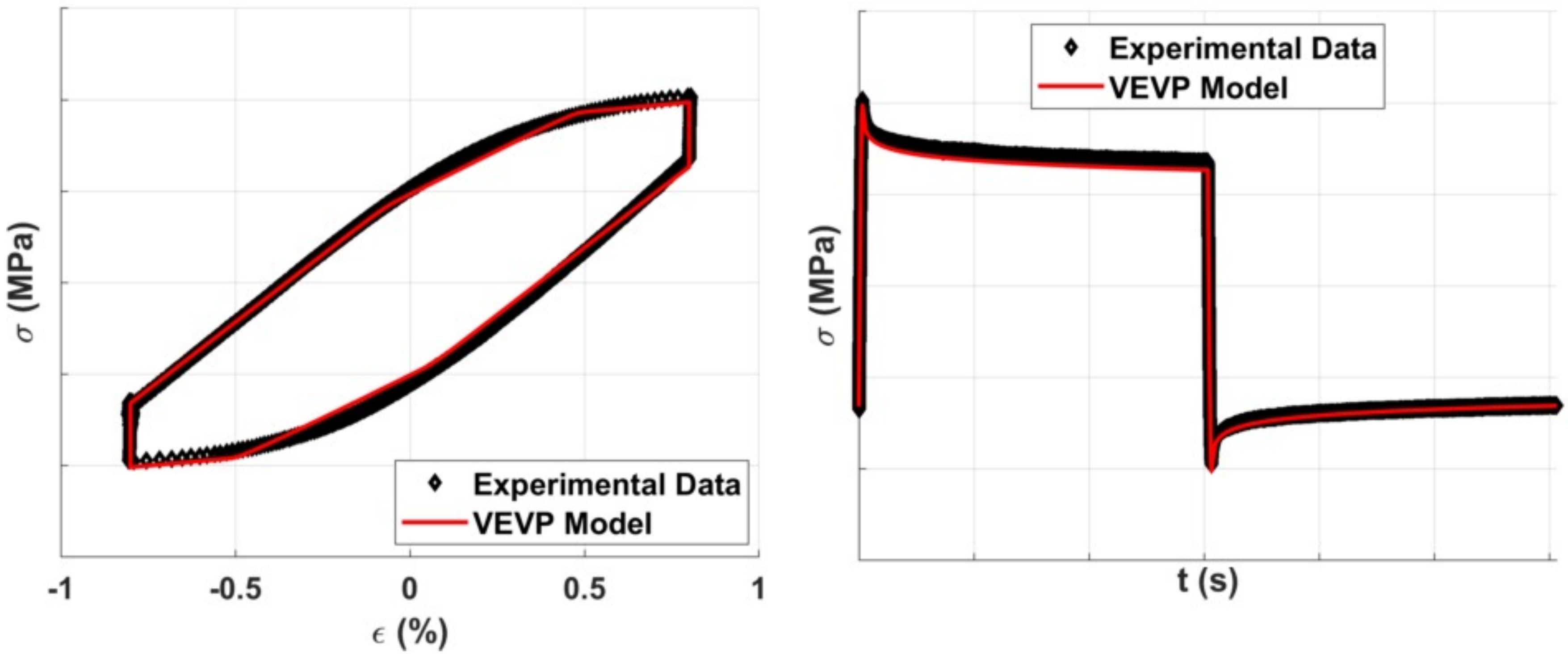
750°C Cyclic Data Prediction

Cycle 2



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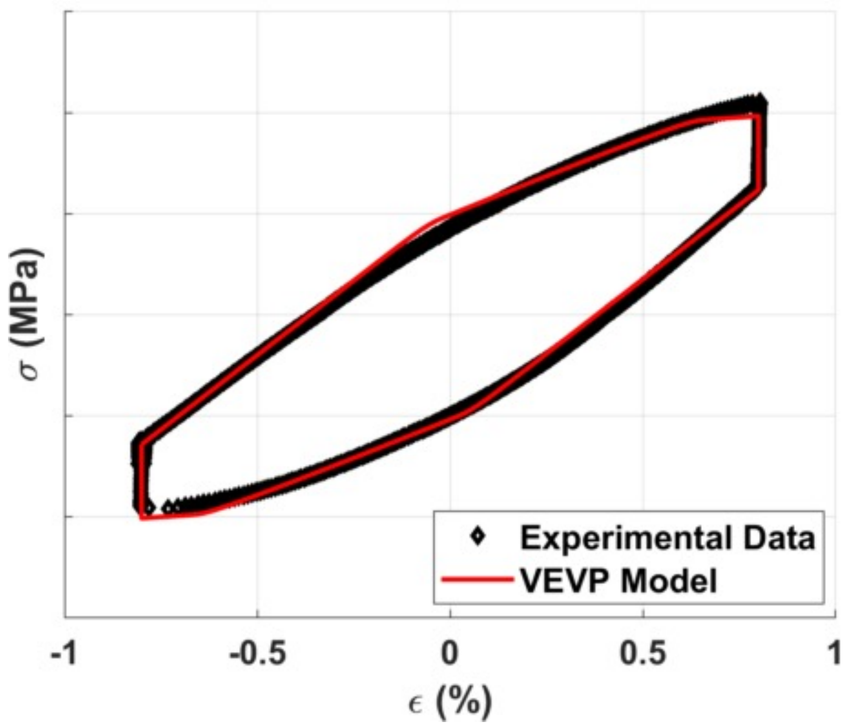
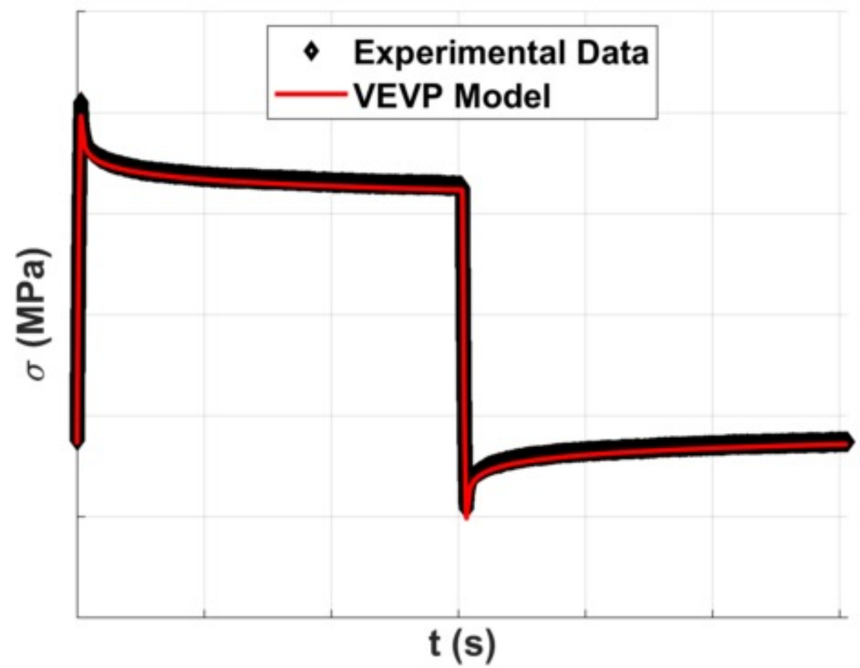
750°C Cyclic Data Prediction

Cycle 10





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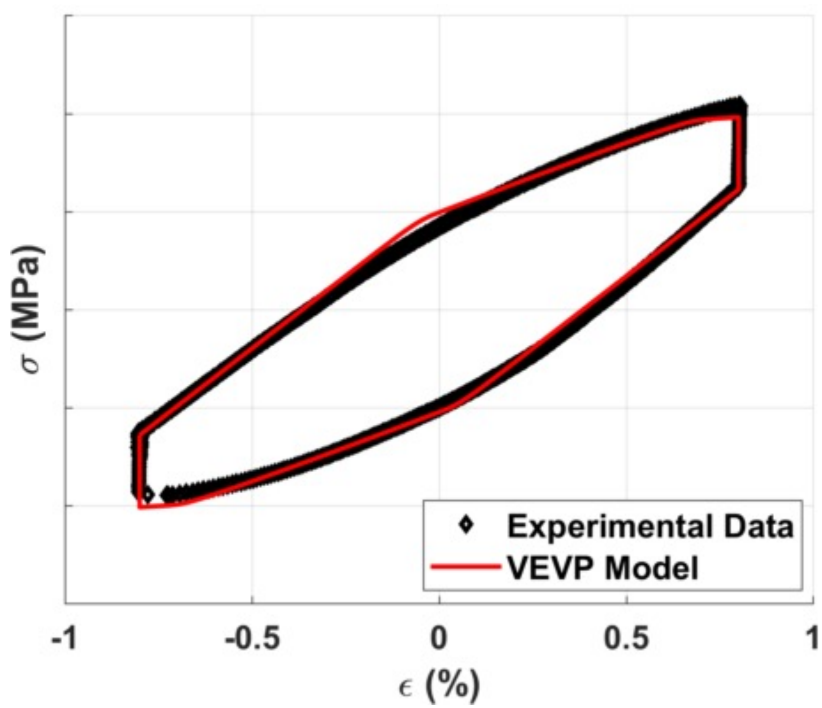
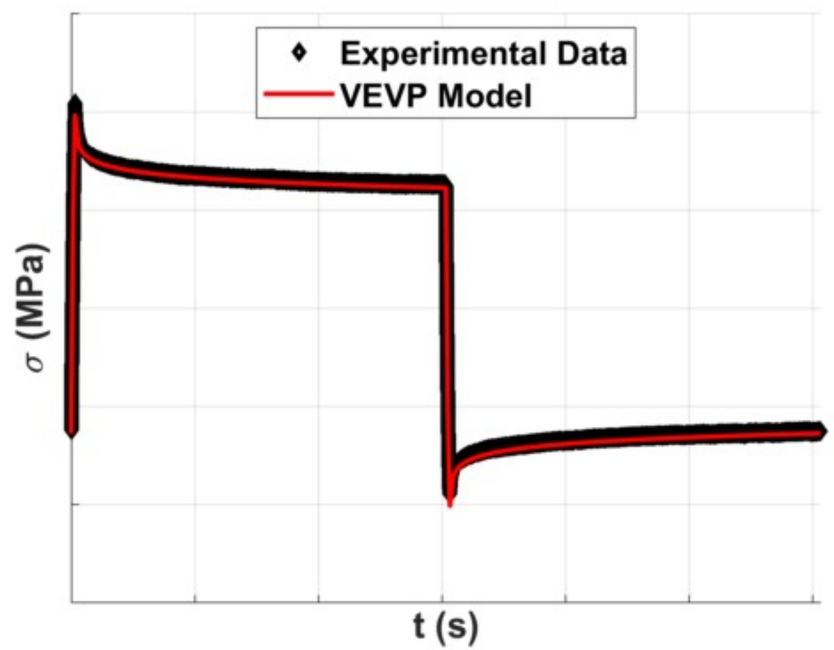
750°C Cyclic Data Prediction

Cycle 15





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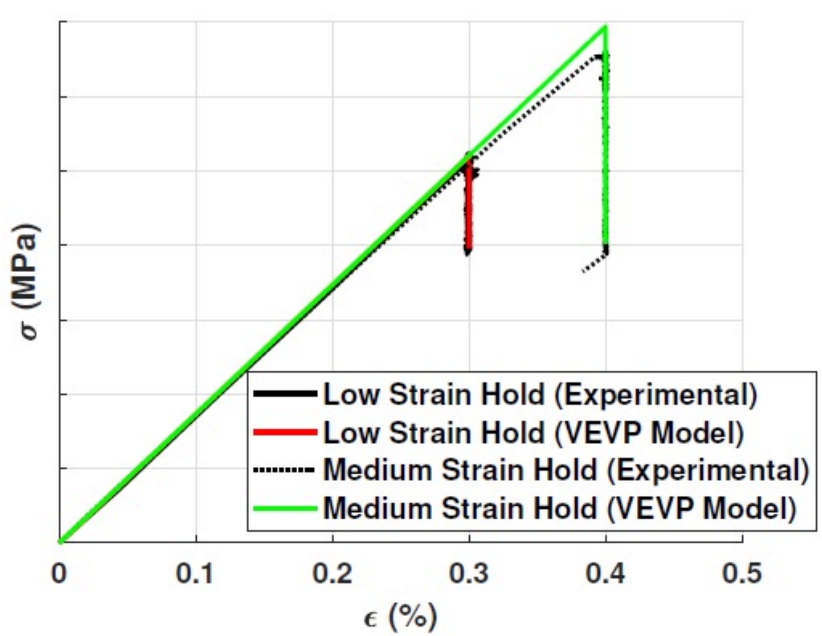
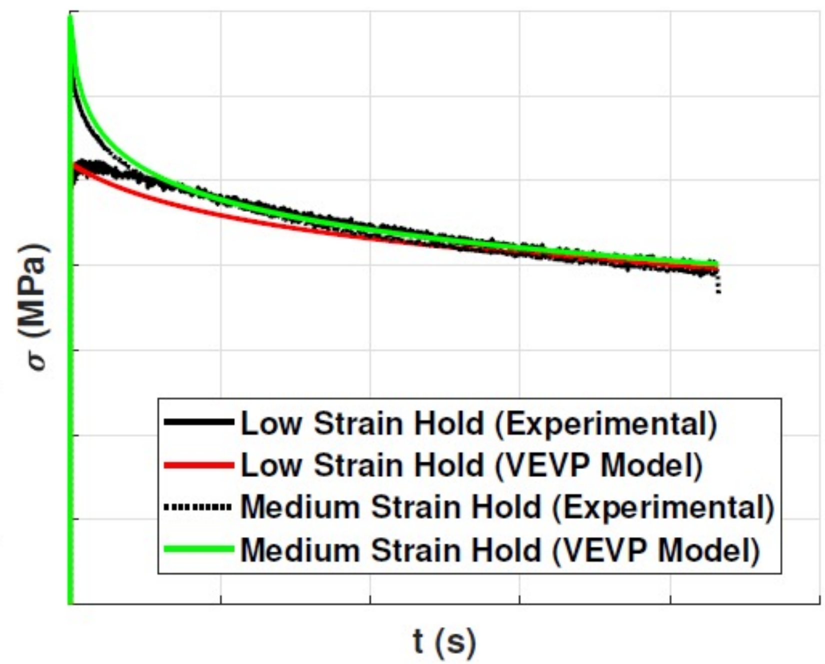
750°C Data Prediction

Stress Relaxation (Parameter Verification)





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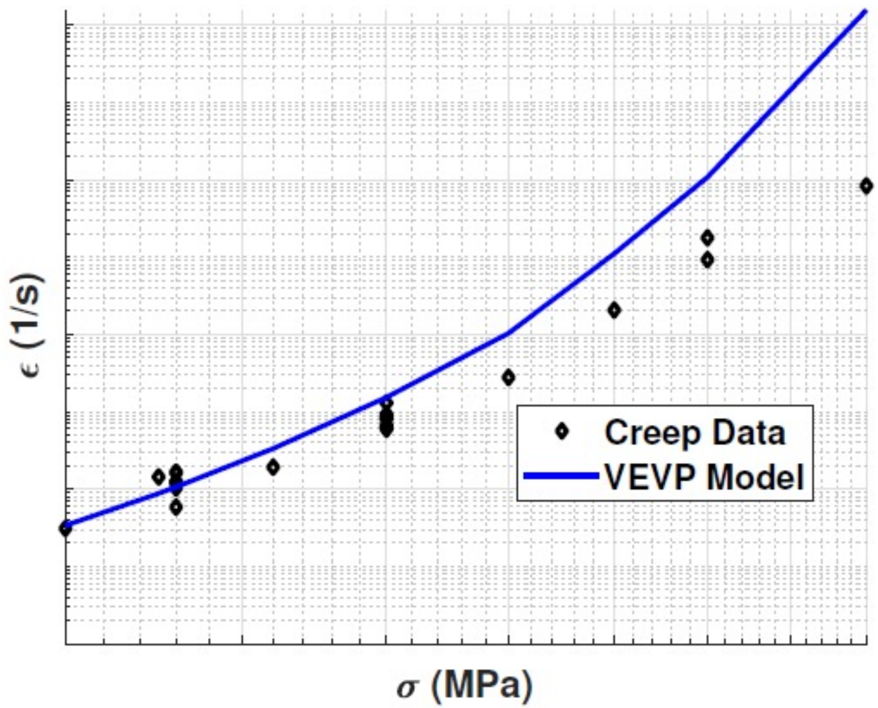
750°C Data Prediction
Creep (Parameter Verification)



This project has received funding from the *European Union's Horizon 2020 research and innovation programme* and Joint Undertaking Clean Sky 2 under grant agreement No 686600.

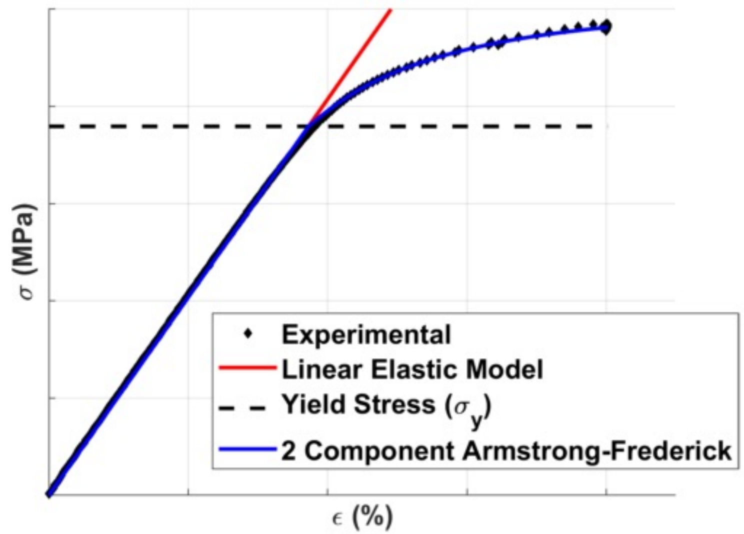
An-isothermal model extensions

Temperature dependent observations

- Temperature rate effects for a generic thermodynamic force **A** (with associated flux **a**) can be determined by taking second derivatives of the free energy.
- Armstrong-Frederick back stress functions may be extended to consider an-isothermal loadings as shown.
- Temperature dependent material parameters will need to be evaluated at instantaneous temperatures, some material parameters an evaluation of the derivative of the related temperature dependent function will also be required.

$$\dot{A} = \frac{\partial^2 \psi}{\partial a_k a_j}\dot{a}_j + \frac{\partial^2 \psi}{\partial a_k T}\dot{T}$$

$$\dot{\chi}_i^\beta = \frac{2}{3}C_i^\beta \dot{\epsilon}_\beta - \gamma_i^\beta \chi_i^\beta \dot{\lambda}_\beta + \frac{1}{C_i^\beta}\frac{\partial C_i^\beta}{\partial T}\dot{T}$$



DevTMF
Clean Sky

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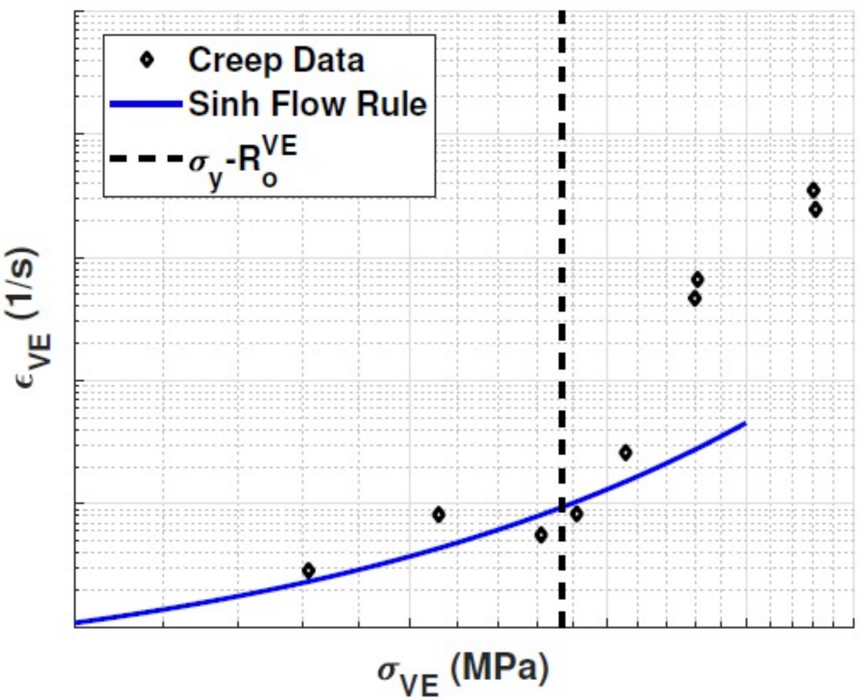
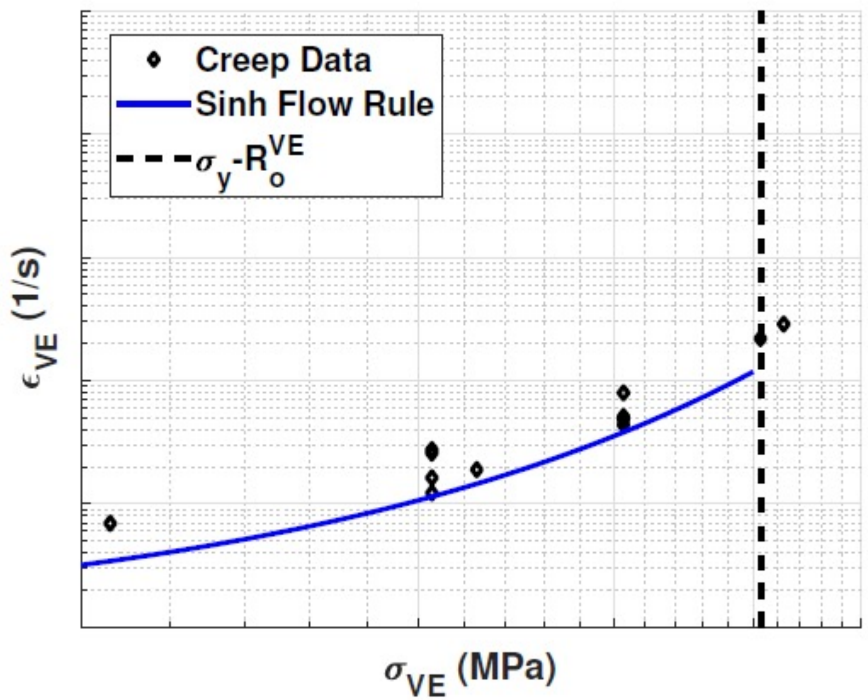
An-isothermal model extensions

Temperature dependent observations





DevTMF

Clean Sky

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Conclusions

Future Work

- A unified viscoelastic - viscoplastic material model has been developed here for the description of cyclic plasticity in RR1000 at 750°C. Extensions to the model are also proposed to include an-isothermal effects.
- The inclusion of Viscoelasticity allows stress relaxation a low loads to be approximated, which would not normally be possible for elastic - viscoplastic model formulations.
- The definition of viscoelastic strain in the presented material model offers exciting opportunities for strain partitioning failure models, for example. Application of such lifing models with additional viscoelastic strain amplitude components will be investigated in future work.

DevTMF

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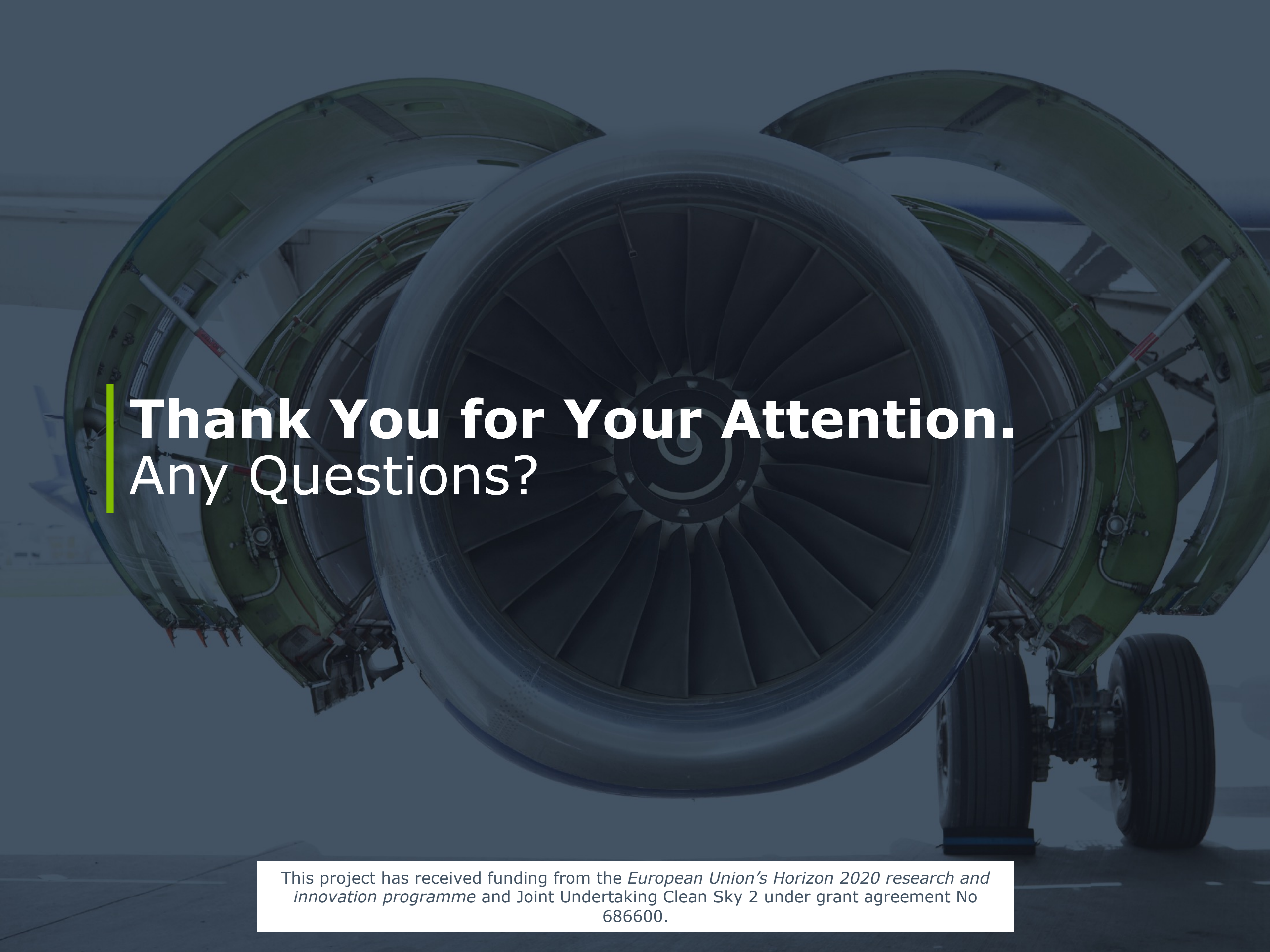

Thank You for Your Attention.
Any Questions?

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